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Strange axial-vector mesons mixing angle

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Abstract. The masses of $K_1(^3P_1)$ and $K_1(^1P_1)$ are considered in a nonrelativistic constituent quark model, and the absolute value of the $K_1(^3P_1)$-$K_1(^1P_1)$ mixing angle is determined to be about 59.29°. The comparison of the theoretical predictions on the strong decay widths of $K_1(1270)$ and $K_1(1400)$ in the $^3P_0$ decay model as well as the production ratio of these two states in the τ decay between the available experimental data strongly favors that the $K_1(^3P_1)$-$K_1(^1P_1)$ mixing angle is about +59.29°.

PACS. 14.40.Ev Other strange mesons – 12.39.Jh Nonrelativistic quark model – 13.25.-k Hadronic decays of mesons – 13.35.Dx Decays of taus

1 Introduction

The strange axial vector mesons provide interesting possibilities to study QCD in the nonperturbative regime by the mixing of the $^3P_1$ and $^1P_1$ states. In the exact $SU(3)$ limit, $K_1(^3P_1)$ and $K_1(^1P_1)$ do not mix, just as the a_1 and b_1 mesons do not mix. For the strange quark mass greater than the up- and down-quark masses so that $SU(3)$ is broken, also, $K_1(^3P_1)$ and $K_1(^1P_1)$ do not possess definite C-parity, therefore these states can in principle mix to give the physical $K_1(1270)$ and $K_1(1400)$.

Accurate determination of θ_K, the mixing angle of $K_1(^3P_1)$ and $K_1(^1P_1)$, is important for comparing the theory predictions about the decays involving the strange axial-mesons with the experimental data. In the literature, θ_K has been estimated by some different approaches, however, there is not yet a consensus on the value of θ_K. As optimum fit to the data as of 1977, Carnegie *et al.* find $\theta_K = (41 \pm 4)^\circ$ [1]. Within the heavy-quark effective theory Isgur and Wise predict two possible mixing angles, $\theta_K \sim 35.3^\circ$ and $\theta_K \sim -54.7^\circ$ [2]. Based on the analysis of $\tau \to \nu K_1(1270)$ and $\tau \to \nu K_1(1400)$, Rosner suggests $\theta_K \sim 62^\circ$ [3], Asner *et al.* give $\theta_K = (69 \pm 16 \pm 19)^\circ$ or $(49 \pm 16 \pm 19)^\circ$ [4], and Cheng obtains $\theta_K = \pm 37^\circ$ or $\pm 58^\circ$ [5]. From the experimental information on masses and the partial rates of $K_1(1270)$ and $K_1(1400)$, Suzuki finds two possible solutions with a two-fold ambiguity, $\theta_K \sim 33^\circ$ or 57° [6]. A constraint $35^\circ \le \theta_K \le 55^\circ$ is predicted by Burakovsky *et al.* in a nonrelativistic constituent quark model [7], and within the same model, the values of $\theta_K \simeq (31 \pm 4)^\circ$ and $\theta_K \simeq (37.3 \pm 3.2)^\circ$ are suggested by Chliapnikov [8] and Burakovsky [9], respectively. The calculations for the strong decays of $K_1(1270)$ and $K_1(1400)$ in the $^3P_0$ decay model suggest $\theta_K \sim 45^\circ$ [10, 11]. The mixing angles $\theta_K \sim 34^\circ$ [12], $\theta_K \sim 5^\circ$ [13] are also presented within a relativized quark model. Vijande *et al.* suggest $\theta_K \sim 55.7^\circ$ based on the calculations in a constituent quark model [14]. More recently, based on the $f_1(1285)$-$f_1(1420)$ mixing angle $\sim 50^\circ$ derived from the analysis for a substantial body of data concerning the $f_1(1420)$ and $f_1(1285)$ [15], we suggest that the $K_1(^3P_1)$-$K_1(^1P_1)$ mixing angle is about $\pm(59.55 \pm 2.81)^\circ$ [16].

In the present work, we shall show that the $K_1(^3P_1)$-$K_1(^1P_1)$ mixing angle derived from the nonrelativistic constituent quark model is in good agreement with that given by ref. [16], and try to constrain the sign of the $K_1(^3P_1)$-$K_1(^1P_1)$ mixing angle by considering the open-flavor strong decays of $K_1(1270)$ and $K_1(1400)$ in the $^3P_0$ decay model and the production ratio of these two states in the τ decay.

2 Nonrelativistic constituent quark model for P-wave mesons

In the constituent quark model, the conventional $q\bar{q}$ wave function is typically assumed to be a solution of a nonrelativistic Schrödinger equation with the generalized Breit-Fermi Hamiltonian which contains a QCD-inspired potential $V(\mathbf{r})$ [17]. The phenomenological forms of the matrix element of the Breit-Fermi Hamiltonian for the $q\bar{q}$ mesons

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Table 1. Angular-momentum part of the matrix elements of (1) and (2).

	$^3P_2$	$^3P_1$	$^3P_0$	$^1P_1$	$^3S_1$	$^1S_0$
$\langle \mathbf{s}_q \cdot \mathbf{s}_{\bar{q}} \rangle$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{3}{4}$	$\frac{1}{4}$	$-\frac{3}{4}$
$\langle \mathbf{L} \cdot \mathbf{S} \rangle$	1	−1	−2	0		
$\langle \mathbf{S}_{q\bar{q}} \rangle$	$-\frac{2}{5}$	2	−4	0		
$\langle \mathbf{L} \cdot \mathbf{S}_- \rangle$	0	0	0	$\frac{3}{2}$		

with orbital angular momentum L are given by [8,18]:

$$M_{L=0} = m_q + m_{\bar{q}} + e_0 \frac{\langle \mathbf{s}_q \cdot \mathbf{s}_{\bar{q}} \rangle}{m_q m_{\bar{q}}}, \quad (1)$$

$$\begin{aligned} M_{L\neq 0} &= m_q + m_{\bar{q}} + a_L + b_L \left(\frac{1}{m_q} + \frac{1}{m_{\bar{q}}}\right) \\ &+ c_L \left(\frac{1}{m_q^2} + \frac{1}{m_{\bar{q}}^2}\right) + \frac{d_L}{m_q m_{\bar{q}}} \\ &+ e_L \frac{\langle \mathbf{s}_q \cdot \mathbf{s}_{\bar{q}} \rangle}{m_q m_{\bar{q}}} + f_L \left(\frac{1}{m_q^3} + \frac{1}{m_{\bar{q}}^3}\right) \\ &+ g_L \left[\frac{(m_q + m_{\bar{q}})^2 + 2 m_q m_{\bar{q}}}{4 m_q^2 m_{\bar{q}}^2} \langle \mathbf{L} \cdot \mathbf{S} \rangle - \frac{m_q^2 - m_{\bar{q}}^2}{4 m_q^2 m_{\bar{q}}^2} \langle \mathbf{L} \cdot \mathbf{S}_- \rangle\right] \\ &+ \frac{h_L}{m_q m_{\bar{q}}} \langle \mathbf{S}_{q\bar{q}} \rangle, \end{aligned} \quad (2)$$

where m_q and $m_{\bar{q}}$ are the constituent quark masses, $\mathbf{s}_q$ and $\mathbf{s}_{\bar{q}}$ are the constituent quark spins, e_0, a_L, b_L, c_L, d_L, e_L, f_L, g_L and h_L are constants, $\mathbf{S} = \mathbf{s}_q + \mathbf{s}_{\bar{q}}$, $\mathbf{S}_- = \mathbf{s}_q - \mathbf{s}_{\bar{q}}$, and $\mathbf{S}_{q\bar{q}} = 3\frac{(\mathbf{s}_q \cdot \mathbf{r})(\mathbf{s}_{\bar{q}} \cdot \mathbf{r})}{r^2} - \mathbf{s}_q \cdot \mathbf{s}_{\bar{q}}$. The angular-momentum part of the matrix elements of (1) and (2) is shown in table 1.

With the help of table 1, applying (1) and (2) to S-wave and P-wave mesons, in the $SU(2)$ flavor symmetry limit, one can obtain[1]

$$\frac{M_\pi + 3M_\rho}{2M_K + 6M_{K^*} - M_\pi - 3M_\rho} = \frac{m_u}{m_s} = 0.6298 \pm 0.00068, \quad (3)$$

and

$$\frac{M(^3P_2)_{s\bar{s}} - M(^1P_1)_{s\bar{s}}}{M(^3P_2)_{n\bar{n}} - M(^1P_1)_{n\bar{n}}} = \frac{m_u^2}{m_s^2}. \quad (4)$$

From (4), with the help of the Gell-Mann–Okubo mass formula [20]

$$M^2(^3P_2)_{s\bar{s}} + M^2(^3P_2)_{n\bar{n}} = 2M^2_{K(^3P_2)}, \quad (5)$$

$$M^2(^1P_1)_{s\bar{s}} + M^2(^1P_1)_{n\bar{n}} = 2M^2_{K_1(^1P_1)}, \quad (6)$$

taking $M(^3P_2)_{n\bar{n}} = M_{a_2(1320)} = 1318.3 \pm 0.6\,\mathrm{MeV}$, $M(^1P_1)_{n\bar{n}} = M_{b_1(1235)} = 1229.5 \pm 3.2\,\mathrm{MeV}$ and $M_{K(^3P_2)} = M_{K_2^*(1430)} = 1429 \pm 0.99\,\mathrm{MeV}$, one can obtain that

$$M_{K_1(^1P_1)} = 1369.52 \pm 1.92\,\mathrm{MeV}. \quad (7)$$

[1] where $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$. All the masses used as input in the present work are taken from PDG [19].

$K_1(^3P_1)$ and $K_1(^1P_1)$ can mix to produce the physical states $K_1(1400)$ and $K_1(1270)$ and the mixing between $K_1(^3P_1)$ and $K_1(^1P_1)$ can be parameterized as [6]

$$\begin{aligned} K_1(1400) &= K_1(^3P_1)\cos\theta_K - K_1(^1P_1)\sin\theta_K, \\ K_1(1270) &= K_1(^3P_1)\sin\theta_K + K_1(^1P_1)\cos\theta_K, \end{aligned} \quad (8)$$

where θ_K denotes the $K_1(^3P_1)$-$K_1(^1P_1)$ mixing angle. Without any assumption about the origin of the $K_1(^3P_1)$-$K_1(^1P_1)$ mixing, the masses of $K_1(^3P_1)$ and $K_1(^1P_1)$ can be related to $M_{K_1(1400)}$ and $M_{K_1(1270)}$, the masses of $K_1(1400)$ and $K_1(1270)$, by the following relation phenomenologically:

$$S \begin{pmatrix} M^2_{K_1(^3P_1)} & A \\ A & M^2_{K_1(^1P_1)} \end{pmatrix} S^\dagger = \begin{pmatrix} M^2_{K_1(1400)} & 0 \\ 0 & M^2_{K_1(1270)} \end{pmatrix}, \quad (9)$$

where A denotes a parameter describing the $K_1(^3P_1)$-$K_1(^1P_1)$ mixing, and

$$S = \begin{pmatrix} \cos\theta_K & -\sin\theta_K \\ \sin\theta_K & \cos\theta_K \end{pmatrix}.$$

From (9), one can have

$$M^2_{K_1(^3P_1)} = M^2_{K_1(1400)}\cos^2\theta_K + M^2_{K_1(1270)}\sin^2\theta_K, \quad (10)$$

$$M^2_{K_1(^1P_1)} = M^2_{K_1(1400)}\sin^2\theta_K + M^2_{K_1(1270)}\cos^2\theta_K, \quad (11)$$

$$\cos(2\theta_K) = \frac{M^2_{K_1(^3P_1)} - M^2_{K_1(^1P_1)}}{M^2_{K_1(1400)} - M^2_{K_1(1270)}}. \quad (12)$$

Inputting $M_{K_1(1400)} = 1402 \pm 7\,\mathrm{MeV}$, $M_{K_1(1270)} = 1273 \pm 7\,\mathrm{MeV}$ and $M_{K_1(^1P_1)} \simeq 1369.52 \pm 1.92\,\mathrm{MeV}$ shown in (7), from (10)-(12), we have

$$\begin{aligned} M_{K_1(^3P_1)} &= 1307.88 \pm 10.33\,\mathrm{MeV}, \\ \theta_K &= \pm(59.29 \pm 2.87)^\circ. \end{aligned} \quad (13)$$

Obviously, the present result regarding $(M_{K_1(^1P_1)}, M_{K_1(^3P_1)}) = (1369.5 \pm 1.92, 1307.88 \pm 10.33)\,\mathrm{MeV}$ and $\theta_K = \pm(59.29 \pm 2.87)^\circ$ is in good agreement with that of $(M_{K_1(^1P_1)}, M_{K_1(^3P_1)}) = (1370.03 \pm 9.69, 1307.35 \pm 0.63)\,\mathrm{MeV}$ and $\theta_K = \pm(59.55 \pm 2.81)^\circ$ given by ref. [16] based on the $f_1(1285)$-$f_1(1420)$ mixing angle $\sim 50^\circ$ extracted from the analysis for a substantial body of data concerning $f_1(1420)$ and $f_1(1285)$ [15].

Within the nonrelativistic constituent quark model, the results regarding the masses of $K_1(^1P_1)$ and $K_1(^3P_1)$, $(M_{K_1(^1P_1)}, M_{K_1(^3P_1)}) = (1368, 1306)\,\mathrm{MeV}$ suggested by [8] and $(M_{K_1(^1P_1)}, M_{K_1(^3P_1)}) = (1356, 1322)\,\mathrm{MeV}$ suggested by [9], are in good agreement with our predicted result. However, based on the following relation employed by [8,9]:

$$\tan^2(2\theta_K) = \left(\frac{M^2_{K_1(^3P_1)} - M^2_{K_1(^1P_1)}}{M^2_{K_1(1400)} - M^2_{K_1(1270)}}\right)^2 - 1, \quad (14)$$

the values of $\theta_K = (31 \pm 4)^\circ$ given by [8] and $\theta_K = (37.3 \pm 3.2)^\circ$ given by [9] disagree with the value of $|\theta_K| \simeq (59.29 \pm 2.87)^\circ$ given by the present work.

As pointed out by our previous paper [16], (14) is equivalent to (12), and will yield two solutions $|\theta_K|$ and $\frac{\pi}{2} - |\theta_K|$. Simultaneously, considering the relations (10), (11) and (14), in the presence of $M_{K_1(1400)} > M_{K_1(1270)}$, we can conclude that if $M_{K_1(^3P_1)} < M_{K_1(^1P_1)}$, $|\theta_K|$ would be greater than 45°. In fact, relation (12) clearly indicates that in the presence of $M_{K_1(1400)} > M_{K_1(1270)}$, the case $M_{K_1(^3P_1)} < M_{K_1(^1P_1)}$ must require $45^\circ < |\theta_K| < 90^\circ$.

3 The sign of θ_K constrained by experimental information

Now we wish to discuss the sign of θ_K by considering the open-flavor strong decays of $K_1(1270)$ and $K_1(1400)$ in the $^3P_0$ decay model, and the production ratio of these two physical strange states in the τ decay.

3.1 Strong decays of $K_1(1270)$ and $K_1(1400)$ in the $^3P_0$ model

The main assumption of the $^3P_0$ decay model is that strong decays take place via the production of a quark-antiquark pair with the vacuum quantum numbers which corresponds to the $^3P_0$ state of a quark-antiquark pair. After the $^3P_0$ decay model was originally introduced by Micu [21], it was applied extensively to meson and baryon decays. It is widely accepted that the $^3P_0$ model is successful since it gives a good description of many of the observed decay amplitudes and partial widths of the open-flavor meson strong decays.

Assuming a fixed $^3P_0$ source strength, simple harmonic-oscillator quark model meson wave functions and a physical phase space, Ackleh *et al.* [22] developed a diagrammatic, momentum-space formulation of the $^3P_0$ model to evaluate the partial width $\Gamma_{A\to BC}$:

$$\Gamma_{A\to BC} = 2\pi \frac{PE_B E_C}{M_A} \sum_{LS} |\mathcal{M}_{LS}|^2, \qquad (15)$$

where P is the decay momentum for the decay $A \to B+C$, E_B and E_C are the energies of mesons B and C, in the rest frame of A,

$$P = \frac{\left[\left(M_A^2 - (M_B+M_C)^2\right)\left(M_A^2 - (M_B-M_C)^2\right)\right]^{1/2}}{2M_A},$$
$$E_B = \left(M_A^2 - M_C^2 + M_B^2\right)/2M_A,$$
$$E_C = \left(M_A^2 - M_B^2 + M_C^2\right)/2M_A,$$

M_A, M_B and M_C denote the masses of the mesons A, B and C, respectively; $\mathcal{M}_{LS}$ are proportional to an overall Gaussian in $x = P/\beta$ times a channel-dependent polynomial $\mathcal{P}_{LS}$,

$$\mathcal{M}_{LS} = \frac{\gamma}{\pi^{1/4}\beta^{1/2}} \mathcal{P}_{LS}(x) e^{-x^2/12}.$$

It is found that this formulation with the width parameter $\beta = 0.4\,\text{GeV}$ and the pair-production strength parameter $\gamma = 0.4$ can give a reasonably accurate description of the overall scale of decay widths [23,24].

Based on (8) and (15), employing the analytical results for $\mathcal{P}_{LS}$ listed in appendix A of ref. [23], one can have [24]

$$\Gamma(K_1(1270)\to\rho K) = 21.8\cos^2\theta_K + 61.6\sin\theta_K\cos\theta_K + 43.6\sin^2\theta_K, \qquad (16)$$

$$\Gamma(K_1(1270)\to\pi K^*) = 59.6\cos^2\theta_K - 158.7\sin\theta_K\cos\theta_K + 115.7\sin^2\theta_K, \qquad (17)$$

$$\Gamma_{\text{thy}}(K_1(1270)) = 81\cos^2\theta_K - 97\sin\theta_K\cos\theta_K + 159\sin^2\theta_K, \qquad (18)$$

$$\Gamma(K_1(1400)\to\rho K) = 160\cos^2\theta_K - 219.9\sin\theta_K\cos\theta_K + 82.3\sin^2\theta_K, \qquad (19)$$

$$\Gamma(K_1(1400)\to\omega K) = 52.3\cos^2\theta_K - 72.3\sin\theta_K\cos\theta_K + 26.8\sin^2\theta_K, \qquad (20)$$

$$\Gamma(K_1(1400)\to\pi K^*) = 141.1\cos^2\theta_K + 176.2\sin\theta_K\cos\theta_K + 78.8\sin^2\theta_K, \qquad (21)$$

$$\Gamma_{\text{thy}}(K_1(1400)) = 353\cos^2\theta_K - 116\sin\theta_K\cos\theta_K + 188\sin^2\theta_K, \qquad (22)$$

$$|D/S|^2 = \begin{cases} \frac{(-0.0411\cos\theta_K - 0.029\sin\theta_K)^2}{(-0.204\cos\theta_K + 0.288\sin\theta_K)^2}, & \text{for } K_1(1270)\to\pi K^*, \\ \frac{(-0.0498\cos\theta_K + 0.0704\sin\theta_K)^2}{(+0.247\cos\theta_K + 0.175\sin\theta_K)^2}, & \text{for } K_1(1400)\to\pi K^*. \end{cases} \qquad (23)$$

For $\theta_K = \pm(59.29 \pm 2.87)^\circ$, the theoretical results regarding the above widths are shown in tables 2, 3 and 4. Tables 2-4 clearly indicate that the present experimental data strongly prefer $\theta_K = +(59.29 \pm 2.87)^\circ$ over $\theta_K = -(59.29 \pm 2.87)^\circ$.

3.2 Production ratio of the $K_1(1270)$ and $K_1(1400)$ in the τ decay

With the definition of the decay constant of the axial-vector meson given by [5]

$$\langle 0|A_\mu|A(q,\varepsilon)\rangle = f_A M_A \varepsilon_\mu, \qquad (24)$$

the partial width for $\tau \to \nu_\tau K_1$ can be expressed by

$$\Gamma(\tau\to\nu_\tau K_1) = \frac{G_F^2}{16\pi}|V_{us}|^2 f_{K_1}^2 \frac{\left(M_\tau^2 + 2M_{K_1}^2\right)\left(M_\tau^2 - M_{K_1}^2\right)^2}{M_\tau^3}. \qquad (25)$$

Considering the $SU(3)$ breaking corrections, following refs. [5,6], we have

$$\frac{f_{K_1(1270)} M_{K_1(1270)}}{f_{K_1(1400)} M_{K_1(1400)}} = \frac{\sin\theta_K - \delta\cos\theta_K}{\cos\theta_K + \delta\sin\theta_K}, \qquad (26)$$

where the parameter δ denoting a $SU(3)$ breaking factor has the following form in the static limit of the quark

Table 2. The predicted results of the $K_1(1270)$ strong decays in the $^3P_0$ decay model. Boldface values stand for experimental results.

$K_1(1270)$	Exp. [19]	$\theta_K = +(59.29 \pm 2.87)^\circ$	$\theta_K = -(59.29 \pm 2.87)^\circ$
Γ (MeV)	$\mathbf{90 \pm 20}$	96.07 ± 5.76	181.25 ± 1.11
$\Gamma(\rho K)/\Gamma(\pi K^*)$	$\mathbf{2.625 \pm 0.902}$	2.07 ± 0.41	0.064 ± 0.014

Table 3. The predicted results of the $K_1(1400)$ strong decays in the $^3P_0$ decay model. Boldface values stand for experimental results.

$K_1(1400)$	Exp. [19]	$\theta_K = +(59.29 \pm 2.87)^\circ$	$\theta_K = -(59.29 \pm 2.87)^\circ$
Γ (MeV)	$\mathbf{174 \pm 13}$	180.1 ± 4.48	282.0 ± 10.0
$\Gamma(\rho K)/\Gamma$	$\mathbf{0.03 \pm 0.03}$	0.033 ± 0.01	0.71 ± 0.04
$\Gamma(\omega K)/\Gamma$	$\mathbf{0.01 \pm 0.01}$	0.0095 ± 0.0034	0.23 ± 0.01
$\Gamma(\pi K^*)/\Gamma$	$\mathbf{0.94 \pm 0.06}$	0.96 ± 0.05	0.063 ± 0.006

Table 4. The $|D/S|^2$ ratios for $K_1(1270) \to \pi K^*$ and $K_1(1400) \to \pi K^*$ in the $^3P_0$ model. Boldface values stand for experimental results.

$\lvert D/S\rvert^2$	Exp. [19]	$\theta_K = +(59.29 \pm 2.87)^\circ$	$\theta_K = -(59.29 \pm 2.87)^\circ$
$K_1(1270) \to \pi K^*$	$\mathbf{1.0 \pm 0.7}$	0.1 ± 0.03	0.0001 ± 0.0002
$K_1(1400) \to \pi K^*$	$\mathbf{0.04 \pm 0.01}$	0.02 ± 0.004	12.5 ± 15.6

model [10][2]:

$$\delta = \frac{1}{\sqrt{2}} \frac{m_s - m_u}{m_s + m_u} = 0.16 \pm 0.0003. \quad (27)$$

From (25) and (26), the $K_1(1400)$ and $K_1(1270)$ production ratio in the τ decay can be given by

$$\frac{\Gamma(\tau \to \nu_\tau K_1(1270))}{\Gamma(\tau \to \nu_\tau K_1(1400))} = F_p \left| \frac{\sin\theta_K - \delta\cos\theta_K}{\cos\theta_K + \delta\sin\theta_K} \right|^2, \quad (28)$$

where F_p denotes the phase factor given by

$$F_p = \frac{\left(M_\tau^2 + 2M^2_{K_1(1270)}\right)\left(M_\tau^2 - M^2_{K_1(1270)}\right)^2 M^2_{K_1(1400)}}{\left(M_\tau^2 + 2M^2_{K_1(1400)}\right)\left(M_\tau^2 - M^2_{K_1(1400)}\right)^2 M^2_{K_1(1270)}} = 1.82 \pm 0.086.$$

Then, from (28), one can have

$$\frac{\Gamma(\tau \to \nu_\tau K_1(1270))}{\Gamma(\tau \to \nu_\tau K_1(1400))} = \begin{cases} 2.62 \pm 0.55, & \text{for } \theta_K = +(59.29 \pm 2.87)^\circ, \\ 11.59 \pm 3.43, & \text{for } \theta_K = -(59.29 \pm 2.87)^\circ. \end{cases} \quad (29)$$

Experimentally, $\mathcal{B}(\tau \to \nu_\tau K_1(1270))$ and $\mathcal{B}(\tau \to \nu_\tau K_1(1400))$ have been reported by the TPC/Two-Gamma Collaboration [25] in 1994 and by the ALEPH Collaboration [26] in 1999, respectively. The averaged result of these two collaborations is given by [19]

$$\mathcal{B}(\tau \to \nu_\tau K_1(1270)) = (0.47 \pm 0.11) \times 10^{-2},$$
$$\mathcal{B}(\tau \to \nu_\tau K_1(1400)) = (0.17 \pm 0.26) \times 10^{-2}, \quad (30)$$

which gives

$$\left.\frac{\Gamma(\tau \to \nu_\tau K_1(1270))}{\Gamma(\tau \to \nu_\tau K_1(1400))}\right|_{\text{exp1}} = 2.76 \pm 4.28. \quad (31)$$

This measured result also is in favor of $\theta_K = +(59.29 \pm 2.87)^\circ$ over $\theta_K = -(59.29 \pm 2.87)^\circ$, although the uncertainty of the reported result is large as shown in (31).

Assuming the resonance structure of $\tau^- \to K^-\pi^+\pi^-\nu_\tau$ decays being dominated by the $K_1(1270)$ and $K_1(1400)$ resonances, in 2000, both CLEO Collaboration [4] and OPAL Collaboration [27] have also measured the ratio of $\nu_\tau K_1(1270)$ to $\nu_\tau K_1(1400)$ with the averaged result [19]

$$\frac{\Gamma(\tau \to \nu_\tau K_1(1270))}{\Gamma(\tau \to \nu_\tau K_1(1270)) + \Gamma(\tau \to \nu_\tau K_1(1400))} = 0.69 \pm 0.15, \quad (32)$$

which therefore in turn implies that

$$\left.\frac{\Gamma(\tau \to \nu_\tau K_1(1270))}{\Gamma(\tau \to \nu_\tau K_1(1400))}\right|_{\text{exp2}} = 2.23 \pm 1.56. \quad (33)$$

The comparison of (29) and (33) again shows that the present experimental data strongly prefer $\theta_K = +(59.29 \pm 2.87)^\circ$ over $\theta_K = -(59.29 \pm 2.87)^\circ$.

Based on the $K_1(1400)$ production dominance in the τ decay, Suzuki suggests that the preferred result is $\theta_K \approx 33^\circ$ rather than 57° [6]. However, the recent available experiment data shown in (33) clearly show the $K_1(1270)$ dominance in the τ decay. Consequently, the argument of ruling out $\theta_K \approx 57^\circ$ from the $K_1(1400)$ dominance is therefore no longer valid. The study of hadronic decays $D \to K_1(1270)\pi$, and $K_1(1400)\pi$ decays performed

[2] $m_u = 307.8 \pm 0.19$ MeV and $m_s = 488.69 \pm 0.28$ MeV derived from (1).

by Cheng [5] favors $\theta_K \approx -58°$, however, as pointed out by Cheng *et al.* in ref. [28], this argument is subject to many uncertainties such as the unknown $D \to K_1(^1P_1), K_1(^3P_1)$ transition form factors and the decay constants of $K_1(1270)$ and $K_1(1400)$. We note that the recent analysis for the $SU(3)$ nonets of the axial vector mesons into a vector and a pseudoscalar performed by Roca *et al.* [29] based on a tensor formulation of the vector and axial vector fields gives $\theta_K = +(62 \pm 3)°$, which is in fact in good agreement with our suggested result that $\theta_K = +(59.29 \pm 2.87)°$.

4 Concluding remarks

In the nonrelativistic constituent quark model, the masses of $K_1(^3P_1)$ and $K_1(^1P_1)$ are determined to be 1307.88 ± 10.33 and 1396.5 ± 1.92 MeV, respectively, which therefore suggests that the absolute value of the $K_1(^3P_1)$-$K_1(^1P_1)$ mixing angle is $(59.29 \pm 2.87)°$. These findings are in good agreement with those given by ref. [16] based on the investigation on the implication of the $f_1(1285)$-$f_1(1420)$ mixing for the $K_1(^3P_1)$-$K_1(^1P_1)$ mixing angle. Investigating the open-flavor strong decays of $K_1(1270)$ and $K_1(1400)$ in the $^3P_0$ decay model, we find that the current experimental data strongly prefer $\theta_K = +(59.29 \pm 2.87)°$ over $\theta_K = -(59.29 \pm 2.87)°$. The analysis for the production ratio of $K_1(1270)$ and $K_1(1400)$ in the τ decay also indicates that the experimental data is in favor of the result $\theta_K = +(59.29 \pm 2.87)°$.

In the framework of a covariant light-front quark model, the calculations performed by Cheng *et al.* [28] for the exclusive radiative B decays, $B \to K_1(1270)\gamma$, $K_1(1400)\gamma$, show that the relative strength of $B \to K_1(1270)\gamma$ and $B \to K_1(1270)\gamma$ rates is very sensitive to the sign of θ_K. The recent analysis of two-body B decays with an axial-vector meson in the final state performed by Nardulli *et al.* [30,31] using naive factorization, shows that the branching ratios for $B \to b_1\pi$, b_1K, $a_1\pi$ and a_1K also depend strongly on θ_K. In addition, as pointed by Suzuki [32], the relation $|Am(J/\psi(\psi') \to K_1^0(1400)\overline{K^0})|^2 = \tan^2\theta_K |Am(J/\psi(\psi') \to K_1^0(1270)\overline{K^0})|^2$ can be able to determine θ_K directly without referring to other parameters. Therefore, in order to further check the consistency of our suggested mixing angle of $K_1(1270)$ and $K_1(1400)$, detailed experimental study of the above-mentioned decays involving the axial-vector mesons is certainly desirable.

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